Coupled oscillators and Feynman's three papers

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Abstract

According to Richard Feynman, the adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing. It is therefore interesting to combine some, if not all, of Feynman's papers into one. The first of his three papers is on the "rest of the universe" contained in his 1972 book on statistical mechanics. The second idea is Feynman's parton picture which he presented in 1969 at the Stony Brook conference on high-energy physics. The third idea is contained in the 1971 paper he published with his students, where they show that the hadronic spectra on Regge trajectories are manifestations of harmonic-oscillator degeneracies. In this report, we formulate these three ideas using the mathematics of two coupled oscillators. It is shown that the idea of entanglement is contained in his rest of the universe, and can be extended to a space-time entanglement. It is shown also that his parton model and the static quark model can be combined into one Lorentz-covariant entity. Furthermore, Einstein's special relativity, based on the Lorentz group, can also be formulated within the mathematical framework of two coupled oscillators.

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1 Introduction

According to Feynman, *the adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.*

According to what he said above, Feynman is saying or at least trying to say the same thing in his numerous papers. Thus, his ultimate goal was to combine all those into one paper. Feynman published about 150 papers. I am not able to combine all those into one paper. However, let us see whether we can combine the following three papers he published during the period 1969-1972.

1. In 1969, Feynman invented partons by observing hadrons moving with velocity close to that of light [1]. Hadrons are collection of partons whose properties are quite different from those of the quarks, invented by Gell-Mann. According to the quark model, hadrons are quantum bound states like the hydrogen atom. While quarks and partons appear differently to us, are they the same covariant entity in different limiting cases?

   This is a Kantian question which Einstein addressed so brilliantly. The energy-momentum relation for slow particles is $E = p^2/2m$, while it is $E = cp$ for fast-moving particles. Einstein showed that they come from the same formula in different limits. His formula was of course $E = \sqrt{(mc^2)^2 + (cp)^2}$.

2. In his paper on harmonic oscillators [2], Feynman notes the existence of Feynman diagrams for tools of quantum mechanics in the relativistic regime. However, for bound-state problems, he suggests that harmonic oscillator could be more effective. Needless to say, he knew that these two methods should solve the same problem of combining quantum mechanics and relativity.

   The 1971 paper Feynman wrote with his students contains many original ideas [2]. However, it is generally agreed that this paper is somewhat short in mathematics. With Marilyn Noz, I wrote a book on this subject, and its title is Theory and Applications of the Poincaré Group. Our earliest paper on this subject was published in 1973.

3. In his book on statistical mechanics [3], Feynman divides the quantum universe into two systems, namely the world in which we do physics,

and the rest of the universe beyond our control. He thinks we can obtain a better understanding of our observable system by considering the universe outside the system. If we assume that the same set of physical laws is applicable to this rest of the universe, Feynman was talking about two entangled systems.

The best way to understand the abstract concept of Feynman's rest of the universe is to use two coupled harmonic oscillators, where one is the world in which we do physics, while the other is beyond our control and thus is the rest of the universe. If those two oscillators are observed by two different observers, the system becomes entangled.

The basic advantage of coupled harmonic oscillators is that the physics is perfectly transparent thanks to mathematical simplicity. What is surprising is that all three of the above subjects can be formulated in terms of two coupled harmonic oscillators. In this way, we are able to combine all three of the above-mentioned research lines into a single physical problem.

In section 2, we discuss the quantum mechanics of two coupled oscillators and write the wave functions in terms of the variables convenient for studying all three of Feynman's papers we intend to discuss in this paper. In section 3, the coupled oscillator system is used to illustrate Feynman's rest of the universe. One of the oscillators corresponds to the world in which we do physics. The other is in the rest of the universe.

In section 4, we elaborate on Feynman's point that, while quantum field theory is effective in solving scattering problems through his Feynman diagrams, it is more convenient to use harmonic oscillators for bound-state problems. Paul A. M. Dirac was quite fond of harmonic oscillators. In section 5, we review his efforts to construct relativistic quantum mechanics using single and couple harmonic oscillators [4]-[7].

In section 6, it is shown possible to construct a Lorentz-covariant harmonic oscillators by combining Dirac's work and the paper Feynman wrote with his students [2]. In section 7, we discuss in detail Feynman's decoherence effect contained in his parton picture.

It is known that Einstein in his early years was influenced by a philosopher named Immanuel Kant [8]. The same thing could appear differently to different observers depending on where they are or how they look at. If Feynman felt that he was looking for the same physics while writing different papers, he was looking for the same thing with different view points. Feynman was a Kantianist.

In the Appendix, we give a physicist's interpretation of Kantianism, and we then conclude that, like Einstein, Feynman was a Kantianist.

2 Coupled Harmonic Oscillators

Let us start with two coupled oscillators described by the Hamiltonian

$$H = \frac{1}{2} \left\{ \frac{1}{m} p_1^2 + \frac{1}{m} p_2^2 + A x_1^2 + A x_2^2 + 2C x_1 x_2 \right\}, \tag{1}$$

where the oscillators are assumed to have the same mass. If we choose coordinate variables

$$z_1 = \frac{1}{\sqrt{2}} (x_1 + x_2),$$

$$z_2 = \frac{1}{\sqrt{2}} (x_1 - x_2), \tag{2}$$

the Hamiltonian can be written as

$$H = \frac{1}{2m} \left\{ p_1^2 + p_2^2 \right\} + \frac{K}{2} \left\{ e^{-2\eta} z_1^2 + e^{2\eta} z_2^2 \right\}, \tag{3}$$

where

$$K = \sqrt{A^2 - C^2}, \qquad \exp(2\eta) = \sqrt{(A - C)/(A + C)}. \tag{4}$$

The eigenfrequencies are $\omega_{\pm} = \omega e^{\pm 2\eta}$ with $\omega = \sqrt{K/m}$.

If y_1 and y_2 are measured in units of $(mK)^{1/4}$, the ground-state wave function of this oscillator system is

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{2} (e^{-2\eta} z_1^2 + e^{2\eta} z_2^2) \right\}, \tag{5}$$

The wave function is separable in the z_1 and z_2 variables. However, for the variables x_1 and x_2, the story is quite different, and can be extended to the issue of entanglement. The key question is how the quantum mechanics in the world of the x_1 variable is affected by the x_2 variable. If there are two separate measurement processes for these variables, these two oscillators are entangled.

Let us write the wave function of equation (5) in terms of x_1 and x_2, then

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{4}\left[e^{-2\eta}(x_1 + x_2)^2 + e^{2\eta}(x_1 - x_2)^2\right]\right\}. \qquad (6)$$

When the system is decoupled with $\eta = 0$, this wave function becomes

$$\psi_0(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{2}(x_1^2 + x_2^2)\right\}. \qquad (7)$$

The system becomes separable and becomes disentangled [9].

The wave functions given in this section are well defined in the present form of quantum mechanics. These wave functions serve as the basic scientific language for all three of Feynman's papers we propose to study in this report.

3 Feynman's Rest of the Universe

In his book on statistical mechanics [3], Feynman makes the following statement about the density matrix. *When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts - the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.*

Feynman then wrote a formula

$$\psi(x, y) = \sum_k C_k(y)\phi_k(x), \qquad (8)$$

which is a complete-set expansion of the wave function ϕ in the orthonormal set of $\phi_k(x)$, but its expansion coefficients $C_k(y)$ depends on another variable y. According to Feynman, the variable x is for the world in which we do physics, y is in the rest of the universe where physics or non-physics is done by a different physicist or a different creature. In this way, Feynman introduced the concept of entanglement.

We can use the coupled oscillators to study Feynman's rest of the universe [9]. In order to accommodate Feynman's original idea more precisely, let us replace x_1 and x_2 in equation (6) by x and y respectively, and write the wave function as

$$\psi_\eta(x,y) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{4}\left[e^{-2\eta}(x+y)^2 + e^{2\eta}(x-y)^2\right]\right\}. \qquad (9)$$

As was discussed in the literature for several different purposes [10, 11, 12], the oscillator wave function of equation (6) can be expanded as

$$\psi_\eta(x,y) = \frac{1}{\cosh\eta} \sum_k (\tanh\eta)^k \phi_k(y)\phi_k(x), \qquad (10)$$

where $\phi_k(x)$ is the normalized harmonic oscillator wave function for the $k-th$ excited state. The coefficient $C_k(y)$ in equation (8) now takes the form

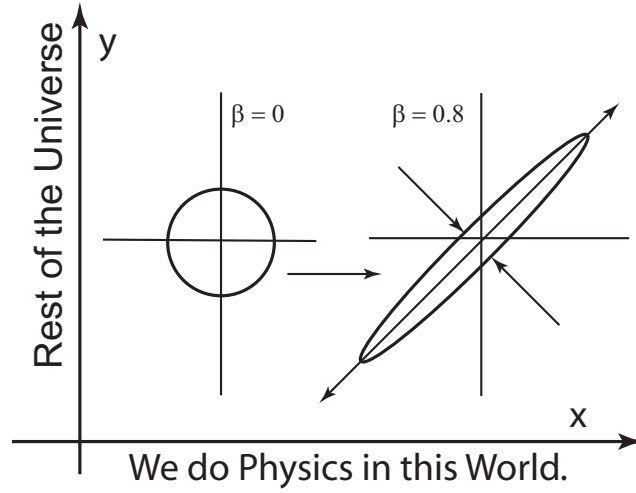$$C_k(y) = \left[\frac{(\tanh\eta)^k}{\cosh\eta}\right]\phi_k(y). \qquad (11)$$

Figure 1: Measurable and non-measurable variables. Here, the x and y variables are the measurable and non-measurable variables. We use β for $\tanh\eta$.

We can use the coupled harmonic oscillators to illustrate what Feynman says in his book. Here we can use respectively x and y for the variable we observe and the variable in the rest of the universe. By using the rest of the universe, Feynman does not rule out the possibility of other creatures measuring the y variable in their part of the universe.

Using the wave function $\psi_\eta(x, y)$ of equation (8), we can construct the pure-state density matrix

$$\rho(x, y; x', y') = \psi_\eta(x, y)\psi_\eta(x', y'), \tag{12}$$

which satisfies the condition $\rho^2 = \rho$:

$$\rho(x, y; x', y') = \int \rho(x, y; z'', y'')\rho(z'', y''; x', y')dx''dy''. \tag{13}$$

If we are not able to make observations on y, we should take the trace of the ρ matrix with respect to the t variable. Then the resulting density matrix is

$$\rho(x, x') = \int \rho(x, y; x', y)dy. \tag{14}$$

The above density matrix can also be calculated from the expansion of the wave function given in equation (10). If we perform the integral of equation (14), the result is

$$\rho(x, x') = \left(\frac{1}{\cosh(\eta)}\right)^2 \sum_k (\tanh\eta)^{2k}\phi_k(x)\phi_k^*(x'). \tag{15}$$

The trace of this density matrix is 1. It is also straightforward to compute the integral for $Tr(\rho^2)$. The calculation leads to

$$Tr\left(\rho^2\right) = \left(\frac{1}{\cosh(\eta)}\right)^4 \sum_k (\tanh\eta)^{4k}. \tag{16}$$

The sum of this series is $1/\cosh(2\eta)$ which is less than one.

This is of course due to the fact that we are averaging over the y variable which we do not measure. The standard way to measure this ignorance is to calculate the entropy defined as

$$S = -Tr\left(\rho\ln(\rho)\right), \tag{17}$$

where S is measured in units of Boltzmann's constant. If we use the density matrix given in equation (15), the entropy becomes

$$S = 2\left\{\cosh^2\eta\ln(\cosh\eta) - \sinh^2\eta\ln(\sinh\eta)\right\}. \qquad (18)$$

This expression can be translated into a more familiar form if we use the notation

$$\tanh\eta = \exp\left(-\frac{\hbar\omega}{kT}\right), \qquad (19)$$

where ω is the unit of energy spacing, and k and T are Boltzmann's constant and absolute Temperature respectively. The ratio $\hbar\omega/kT$ is a dimensionless variable. In terms of this variable, the entropy takes the form [13, 14]

$$S = \left(\frac{\hbar\omega}{kT}\right)\frac{1}{\exp(\hbar\omega/kT) - 1} - \ln\left[1 - \exp(-\hbar\omega/kT)\right]. \qquad (20)$$

This familiar expression is for the entropy of an oscillator state in thermal equilibrium. Thus, for this oscillator system, we can relate our ignorance of the time-separation variable to the temperature. It is interesting to note that the boost parameter or coupling strength measured by η can be related to a temperature variable.

4 Feynman's Oscillators

In his invited talk at the 1970 spring meeting of the American Physical Society held in Washington, DC (U.S.A.), Feynman was discussing hadronic mass spectra and a possible covariant formulation of harmonic oscillators. He noted that the mass spectra are consistent with degeneracy of three-dimensional harmonic oscillators. Furthermore, Feynman stressed that Feynman diagrams are not necessarily suitable for relativistic bound states and that we should try harmonic oscillators. Feynman's point was that, while plane-wave approximations in terms of Feynman diagrams work well for relativistic scattering problems, they are not applicable to bound-state problems. We can summarize what Feynman said in figure 2.

In their 1971 paper [2], Feynman, Kislinger and Ravndal started their harmonic oscillator formalism by defining coordinate variables for the quarks confined within a hadron. Let us use the simplest hadron consisting of two
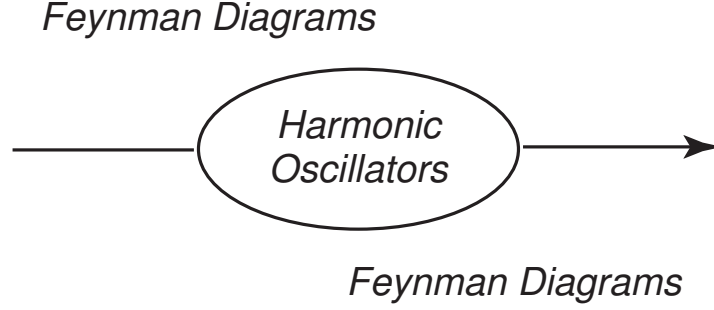
Feynman Diagrams

*Harmonic
Oscillators*

Feynman Diagrams

Figure 2: Feynman's roadmap for combining quantum mechanics with special relativity. Feynman diagrams work for running waves, and they provide a satisfactory resolution for scattering states in Einstein's world. For standing waves trapped inside an extended hadron, Feynman suggested harmonic oscillators as the first step.

quarks bound together with an attractive force, and consider their space-time positions x_a and x_b, and use the variables

$$X = (x_a + x_b)/2, \qquad x = (x_a - x_b)/2\sqrt{2}. \tag{21}$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks. According to Einstein, this space-time separation contains a time-like component which actively participates as in equation (26), if the hadron is boosted along the z direction. This boost can be conveniently described by the light-cone variables defined in Eq(27).

What do Feynman *et al.* say about this oscillator wave function? In their classic 1971 paper [2], they start with the following Lorentz-invariant differential equation.

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x). \tag{22}$$

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. This differential equation gives a complete set of three-dimensional oscillator solutions with which we are familiar in non-relativistic quantum mechanics.

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Indeed, Feynman *et al.* studied in detail the degeneracy of the three-dimensional harmonic oscillators, and compared their results with the observed experimental data for hadrons in the quark model. Their work is complete and thorough, and is consistent with the $O(3)$-like symmetry dictated by Wigner's little group for massive particles [15].

Although this paper contained the above mentioned original ideas of Feynman, it contains some serious mathematical flaws. Feynman *et al.* start with a Lorentz-invariant differential equation for the harmonic oscillator for the quarks bound together inside a hadron. For the two-quark system, they write the wave function of the form

$$\exp\left\{-\frac{1}{2}\left(z^2 - t^2\right)\right\}, \tag{23}$$

where z and t are the longitudinal and time-like separations between the quarks. This form is invariant under the boost, but is not normalizable in the t variable. We do not know what physical interpretation to give to this the above expression.

Yet, Feynman *et al.* make an apology that the symmetry is not $O(3,1)$. This unnecessary apology causes a confusion not only to the readers but also to the authors themselves, and makes the paper difficult to read. Let us see how we can clear up this confusion by looking at what Dirac did with harmonic oscillators.

5 Dirac's Relativistic Quantum Mechanics

I was fortunate enough to have private conversations with Paul A. M. Dirac. In 1962, when he was visiting the University of Maryland, I was a first-year assistant professor, and I had to provide personal convenience to him. I naturally had an occasion to ask him what the most problem in physics was at that time, while Physical Review Letters was carrying articles about Regge poles, bootstraps, and other S-matrix items.

Dirac was telling me that physicists in general do not understand the difference between Lorentz invariance and Lorentz covariance. We said further that we should have a deeper understanding of the covariance if we are to make progress in physics. According to his paper published in 1963 [7], Dirac in 1962 was working on constructing a representation of the $O(3,2)$ group using two coupled oscillators. Since the $S(3,2)$ deSitter group contains the

$O(3,1)$ Lorentz group as a subgroup, Dirac was essentially telling me to do what I am reporting in this paper.

In 1978, I was again able to talk with him while attending one of the Coral Gables conferences held in Miami (Florida). At that time, I was studying his 1949 paper on "Forms of Relativistic Dynamics." In his "instant form," he writes down a formula which could be interpreted as a suppression of time-like oscillations. I asked him whether I could interpret it in my way. His reply was that it depends on how I build the model.

I then pointed out his 1927 paper [4] on the time-energy uncertainty relation and asked him whether I could use his idea to suppress time-like oscillations. Dirac was clearly aware of this paper and mentioned the word "c-number" time-energy uncertainty relation. In consideration of his age, I did not press him any further. Let us see how we could construct a model still within Dirac's framework.

During World War II, Dirac was looking into the possibility of constructing representations of the Lorentz group using harmonic oscillator wave functions [5]. The Lorentz group is the language of special relativity, and the present form of quantum mechanics starts with harmonic oscillators. Presumably, therefore, he was interested in making quantum mechanics Lorentz-covariant by constructing representations of the Lorentz group using harmonic oscillators.

In his 1945 paper [5], Dirac considered the Gaussian form

$$\exp\left\{-\frac{1}{2}\left(x^2 + y^2 + z^2 + t^2\right)\right\}. \tag{24}$$

This Gaussian form is in the $(x,\ y,\ z,\ t)$ coordinate variables. Thus, if we consider Lorentz boost along the z direction, we can drop the x and y variables, and write the above equation as

$$\exp\left\{-\frac{1}{2}\left(z^2 + t^2\right)\right\}. \tag{25}$$

This is a strange expression for those who believe in Lorentz invariance. The expression $(z^2 + t^2)$ is not invariant under Lorentz boost. Therefore Dirac's Gaussian form of equation (25) is not a Lorentz-invariant expression.

On the other hand, this expression is consistent with his earlier papers on the time-energy uncertainty relation [4]. In those papers, Dirac observed that there is a time-energy uncertainty relation, while there are no excitations

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along the time axis. He called this the "c-number time-energy uncertainty" relation.

In 1949, the Reviews of Modern Physics published a special issue to celebrate Einstein's 70th birthday. This issue contains Dirac's paper entitled "Forms of Relativistic Dynamics" [6]. In this paper, he introduced his light-cone coordinate system, in which a Lorentz boost becomes a squeeze transformation.

When the system is boosted along the z direction, the transformation takes the form

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}. \tag{26}$$

The light-cone variables are defined as [6]

$$u = (z+t)/\sqrt{2}, \qquad v = (z-t)/\sqrt{2}, \tag{27}$$

the boost transformation of equation (26) takes the form

$$u' = e^{\eta}u, \qquad v' = e^{-\eta}v. \tag{28}$$

The u variable becomes expanded while the v variable becomes contracted, as is illustrated in figure 3. Their product

$$uv = \frac{1}{2}(z+t)(z-t) = \frac{1}{2}\left(z^2 - t^2\right) \tag{29}$$

remains invariant. In Dirac's picture, the Lorentz boost is a squeeze transformation.

This transformation changes the Gaussian form of equation (25) into

$$\psi_{\eta}(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta}u^2 + e^{2\eta}v^2\right)\right\}, \tag{30}$$

as illustrated in figure 4

Let us go back to section 2 on the coupled oscillators. The above expression is the same as equation (5). The x_1 variable now becames the longitudinal variable z, and x_2 became the time-like variable t.

We can use the coupled harmonic oscillator as the starting point of relativistic quantum mechanics. This allows us to translate the quantum mechanics of two coupled oscillators defined over the space of x_1 and x_2 into quantum mechanics defined over the space-time region of z and t.
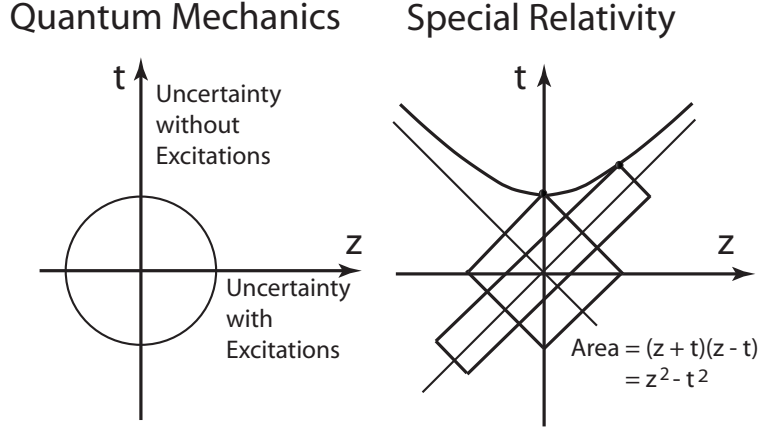
Figure 3: Dirac's form of relativistic quantum mechanics.

This form becomes (25) when η becomes zero. The transition from equation (25) to equation (30) is a squeeze transformation. It is now possible to combine what Dirac observed into a covariant formulation of the harmonic oscillator system. First, we can combine his c-number time-energy uncertainty relation described in figure 3 and his light-cone coordinate system of the same figure into a picture of covariant space-time localization given in figure 4.

The wave function of equation (25) is distributed within a circular region in the uv plane, and thus in the zt plane. On the other hand, the wave function of equation (30) is distributed in an elliptic region with the light-cone axes as the major and minor axes respectively. If η becomes very large, the wave function becomes concentrated along one of the light-cone axes. Indeed, the form given in equation (30) is a Lorentz-squeezed wave function. This squeeze mechanism is illustrated in figure 4.

There are two homework problems which Dirac left us to solve. First, in defining the t variable for the Gaussian form of equation (25), Dirac did not specify the physics of this variable. If it is going to be the calendar time, this form vanishes in the remote past and remote future. We are not dealing with this kind of object in physics. What is then the physics of this time-like t variable?

The Schrödinger quantum mechanics of the hydrogen atom deals with localized probability distribution. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in
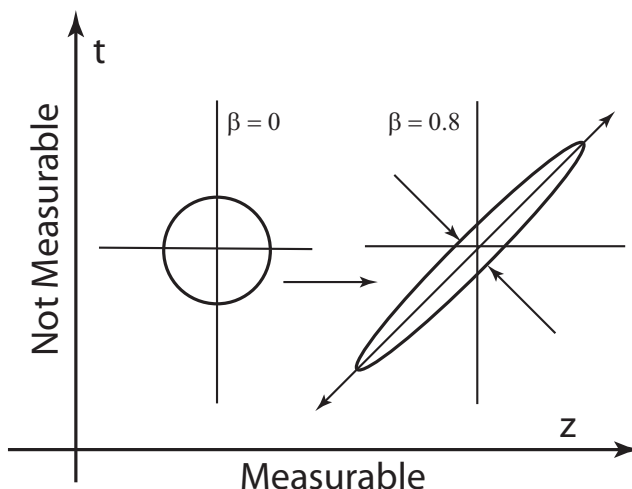
Figure 4: The world in which we do physics and the rest of the universe. This figure is identical to that of figure 1, but the axis labels are different, indicating the physics is different. Yet, the physics of coupled oscillators could be translated into the physics of Lorentzian space and time.

terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be a time-separation between the two constituent particles, and an uncertainty relation applicable to this separation variable. Dirac did not say in his papers of 1927 and 1945, but Dirac's "t" variable is applicable to this time-separation variable. This time-separation variable will be discussed in detail in section 4 for the case of relativistic extended particles.

Second, as for the time-energy uncertainty relation, Dirac'c concern was how the c-number time-energy uncertainty relation without excitations can be combined with uncertainties in the position space with excitations. How can this space-time asymmetry be consistent with the space-time symmetry of special relativity?

Both of these questions can be answered in terms of the space-time symmetry of bound states in the Lorentz-covariant regime [11]. In his 1939

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paper [15], Wigner worked out internal space-time symmetries of relativistic particles. He approached the problem by constructing the maximal subgroup of the Lorentz group whose transformations leave the given four-momentum invariant. As a consequence, the internal symmetry of a massive particle is like the three-dimensional rotation group which does not require transformation into time-like space.

If we extend Wigner's concept to relativistic bound states, the space-time asymmetry which Dirac observed in 1927 is quite consistent with Einstein's Lorentz covariance. Indeed, Dirac's time variable can be treated separately. Furthermore, it is possible to construct a representations of Wigner's little group for massive particles [11]. As for the time-separation which can be linearly mixed with space-separation variables when the system is Lorentz-boosted, it has its role in internal space-time symmetry.

Dirac's interest in harmonic oscillators did not stop with his 1945 paper on the representations of the Lorentz group. In his 1963 paper [7], he constructed a representation of the $O(3,2)$ deSitter group using two coupled harmonic oscillators. This paper contains not only the mathematics of combining special relativity with the quantum mechanics of quarks inside hadrons, but also forms the foundations of two-mode squeezed states which are so essential to modern quantum optics [12, 16, 17]. Dirac did not know this when he was writing this 1963 paper. Furthermore, the $O(3,2)$ deSitter group contains the Lorentz group $O(3,1)$ as a subgroup. Thus, Dirac's oscillator representation of the deSitter group essentially contains all the mathematical ingredients of what we are studying in this paper.

It is also interesting to note that, in addition to Dirac and Feynman, there are other authors who attempted to construct normalizable harmonic oscillators which can be Lorentz-boosted [18]. If the concept of wave functions is to be consistent with Lorentz covariance, the first wave function has to be the harmonic-oscillator wave function.

6 Covariant Oscillators and Entangled Oscillators

The simplest solution to the differential equation of equation (22) takes the form of equation (25). If we allow excitations along the longitudinal coordinate and forbid excitations along the time coordinate, the wave function

takes the form

$$\psi_0^n(z, t) = C_n H_n(z) \exp\left\{-\frac{1}{2}\left(z^2 + t^2\right)\right\}, \tag{31}$$

where H_n is the Hermite polynomial of the n-th order, and C_n is the normalization constant.

If the system is boosted along the z direction, the z and t variables should be replaced by z' and t' respectively with

$$z' = (\cosh \eta)z - (\sinh \eta)t, \qquad t' = (\cosh \eta)t - (\sinh \eta)z. \tag{32}$$

The Lorentz-boosted wave function takes the form

$$\psi_\eta^n(z, t) = H_n(z') \exp\left\{-\frac{1}{2}\left(z'^2 + t'^2\right)\right\}, \tag{33}$$

It is indeed possible to construct the representation of Wigner's $O(3)$-like little group for massive particles using these oscillator solutions [11]. This allows us to use this oscillator system for wave functions in the Lorentz-covariant world.

However, presently, we are interested in space-time localizations of the wave function dictated by the Gaussian factor of the ground-state wave function. In the light-cone coordinate system, the Lorentz-boosted wave function becomes

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta}u^2 + e^{2\eta}v^2\right)\right\}, \tag{34}$$

as given in equation (30). This wave function can be written as

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{4}\left[e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2\right]\right\}. \tag{35}$$

Let us go back to equation (6) for the coupled oscillators. If we replace x_1 and x_2 by z and t respectively, we arrive at the above expression for the covariant harmonic oscillators.

We are of course talking about two different physical systems. For the case of coupled oscillators, there are two one-dimensional oscillators. In the case of covariant harmonic oscillators, there is one oscillator with two variables. The Lorentz boost corresponds to coupling of two oscillators. With these points in mind, we can translate the physics of coupled oscillators into the physics of the covariant harmonic oscillators.

We can obtain the expansion of equation (35) from equation (10) by replacing x and y by z and t respectively, and the expression becomes

$$\psi_\eta(z,t) = \frac{1}{\cosh\eta} \sum_k (\tanh\eta)^k \phi_k(z)\phi_k(t). \tag{36}$$

Thus the space variable z and the time variable t are entangled in the same manner as given in Ref. [19]. However, there is a very important difference. The z variable is well defined in the present form of quantum mechanics, but the time-separation variable t is not. First of all, it is different from the calendar time. Both father and son become old according to the calendar time, but their age difference remains invariant. However, this time separation becomes different in different Lorentz frames, because the simultaneity in special relativity is not an invariant concept [20].

Does this time-separation variable exist when the hadron is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Like the y variable of section 3, this time-separation variable belongs to Feynman's rest of the universe.

Using the wave function $\psi_\eta(z,t)$ of equation (35), we can construct the pure-state density matrix

$$\rho(z,t;z',t') = \psi_\eta(z,t)\psi_\eta(z',t'), \tag{37}$$

which satisfies the condition $\rho^2 = \rho$:

$$\rho(z,t;z',t') = \int \rho(z,t;z'',t'')\rho(z'',t'';z',t')dz''dt''. \tag{38}$$

If we are not able to make observations on t, we should take the trace of the ρ matrix with respect to the t variable. Then the resulting density matrix is

$$\rho(z,z') = \int \rho(z,t;z',t)dt. \tag{39}$$

The above density matrix can also be calculated from the expansion of the wave function given in equation (36). If we perform the integral of equation (14), the result becomes

$$\rho(z,z') = \left(\frac{1}{\cosh(\eta)}\right)^2 \sum_k (\tanh\eta)^{2k} \phi_k(z)\phi_k^*(z'). \tag{40}$$

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which becomes identical to the expression of equation (15) if the variables z and z' are replaced by x and x' respectively. We can then construct the same logic as the one following equation (15) to get the entropy [13].

Let us summarize. At this time, the only theoretical tool available to this time-separation variable is through the space-time entanglement, which generate entropy coming from the rest of the universe. If the time-separation variable is not measured the entropy is one of the variables to be taken into account in the Lorentz-covariant system.

In spite of our ignorance about this time-separation variable from the theoretical point of view, its existence has been proved beyond any doubt in high-energy laboratories. We shall see in section 7 that it plays a role in producing a decoherence effect observed universally in high-energy laboratories.

7   Parton Picture and Decoherence

In a hydrogen atom or a hadron consisting of two quarks, there is a spacial separation between two constituent elements. In the case of the hydrogen atom we call it the Bohr radius. If the atom or hadron is at rest, the time-separation variable does not play any visible role in quantum mechanics. However, if the system is boosted to the Lorentz frame which moves with a speed close to that of light, this time-separation variable becomes as important as the space separation of the Bohr radius. Thus, the time-separation plays a visible role in high-energy physics which studies fast-moving bound states. Let us study this problem in more detail.

It is a widely accepted view that hadrons are quantum bound states of quarks having a localized probability distribution. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra [2, 11]. However, this picture of bound states is applicable only to observers in the Lorentz frame in which the hadron is at rest. How would the hadrons appear to observers in other Lorentz frames?

In 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many "partons" whose properties appear to be quite different from those of the quarks [1]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an ob-

server in a different Lorentz frame? Feynman made the following systematic observations.

a. The picture is valid only for hadrons moving with velocity close to that of light.

b. The interaction time between the quarks becomes dilated, and part ons behave as free independent particles.

c. The momentum distribution of partons becomes widespread as the hadron moves fast.

d. The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears a s a paradox, particularly b) and c) together. How can a free particle have a wide-spread momentum distribution?

In order to resolve this paradox, let us construct the momentum-energy wave function corresponding to equation (30). If the quarks have the four-momenta p_a and p_b, we can construct two independent four-momentum variables [2]

$$P = p_a + p_b, \qquad q = \sqrt{2}(p_a - p_b). \tag{41}$$

The four-momentum P is the total four-momentum and is thus the hadronic four-momentum. q measures the four-momentum separation between the quarks. Their light-cone variables are

$$q_u = (q_0 - q_z)/\sqrt{2}, \qquad q_v = (q_0 + q_z)/\sqrt{2}. \tag{42}$$

The resulting momentum-energy wave function is

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left[e^{-2\eta}q_u^2 + e^{2\eta}q_v^2\right]\right\}. \tag{43}$$

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function of equation (30). The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [11, 21, 22]. The hadronic structure
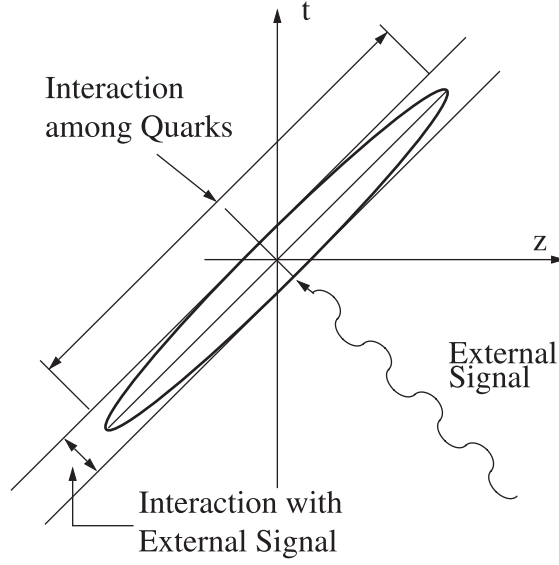
Figure 5: Lorentz-squeezed hadron and interaction times. Quarks interact among themselves and with external signal. The interaction time of the quarks among themselves become dilated, as the major axis of this ellipse indicates. On the other hand, the external signal, since it is moving in the direction opposite to the direction of the hadron, travels along the negative light-cone axis. To the external signal, if it moves with velocity of light, the hadron appears very thin, and the quark's interaction time with the external signal becomes very small.

function calculated from this formalism is in a reasonable agreement with the experimental data [23].

When the hadron is at rest with $\eta = 0$, both wave functions behave like those for the static bound state of quarks. As η increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the z-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function. The longitudinal momentum distribution becomes wide-spread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from nonrelativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a wide-spread distribution.

However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the momentum-energy width increase in the same direction as the hadron is boosted. This is of course an effect of Lorentz covariance. This indeed is to the resolution of one of the quark-parton puzzles [11, 21, 22].

Another puzzling problem in the parton picture is that partons appear as incoherent particles, while quarks are coherent when the hadron is at rest. Does this mean that the coherence is destroyed by the Lorentz boost? The answer is NO, and here is the resolution to this puzzle.

When the hadron is boosted, the hadronic matter becomes squeezed and becomes concentrated in the elliptic region along the positive light-cone axis. The length of the major axis becomes expanded by e^η, and the minor axis is contracted by e^η.

This means that the interaction time of the quarks among themselves become dilated. Because the wave function becomes wide-spread, the distance between one end of the harmonic oscillator well and the other end increases. This effect, first noted by Feynman [1], is universally observed in high-energy hadronic experiments. The period of oscillation is increases like e^η, as indicated in figure 5.

On the other hand, the external signal, since it is moving in the direction opposite to the direction of the hadron travels along the negative light-cone

axis, as is seen in figure 5. If the hadron contracts along the negative light-cone axis, the interaction time decreases by $e^{-\eta}$. The ratio of the interaction time to the oscillator period becomes $e^{-2\eta}$. The energy of each proton coming out of the Fermilab accelerator is $900 GeV$. This leads the ratio to 10^{-6}. This is indeed a small number. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron.

Feynman's parton picture is one concrete physical example where the decoherence effect is observed [24]. As for the entropy, the time-separation variable belongs to the rest of the universe. Because we are not able to observe this variable, the entropy increases as the hadron is boosted to exhibit the parton effect. The decoherence is thus accompanied by an entropy increase.

Let us go back to the coupled-oscillator system. The light-cone variables in equation (30) correspond to the normal coordinates in the coupled-oscillator system given in equation (2). According to Feynman's parton picture, the decoherence mechanism is determined by the ratio of widths of the wave function along the two normal coordinates.

This decoherence mechanism observed in Feynman's parton picture is quite different from other irreversible decoherences discussed in the literature. It is widely understood that the word decoherence is the loss of coherence within a system. On the other hand, Feynman's decoherence discussed in this section comes from the way external signal interacts with the internal constituents.

Concluding Remarks

Modern physics is a physics of harmonic oscillators and/or two-by-two matrices, since otherwise problems are not soluble. Thus, all soluble problems can be combined into one mathematical framework. Richard Feynman was always interested in understanding physical problems with soluble models. It appears that he had in mind models based on two coupled harmonic oscillators.

It is now possible to combine Feynman's three papers into one formalism. The history of physics tells us that new physics comes whenever we combine different theories into one. Maxwell attempted to combine electricity and magnetism, and he ended up with electromagnetic waves. Einstein combined dynamics and electromagnetism into one transformation law. He invented

relativity.

It is interesting to see that the concept of entanglement is contained in his rest of the universe. We see also that the Lorentz-boosted oscillators and coupled oscillators share the same mathematics, and that we can learn properties of Lorentz boosts using coupled oscillators whose physics is very transparent to us.

In addition, using the coupled oscillators, we can clarify the question of whether the quark model and the parton model are two different manifestation of one covariant entity.

It would be interesting to combine more of Fyenman's into the two-oscillator regime which we discussed in the present report. It is also possible to include papers written by others in this system. In fact, in the present report, we included in this regime Dirac's papers quoted in references [4]-[7]. You may also consider including your own papers.

If we assemble those pieces of works done by Feynman and others, we are able to construct a history table described in figure 6. As Feynman stated in 1970, Feynman diagrams are not effective in dealing with bound-state problems in the Lorentz-covariant regime. We should try harmonic oscillators for the bound-state problem. In this report, it was shown possible to construct a covariant picture of bound states.

In so doing, we have not introduced any new physical principles. We used only the existing rules of quantum mechanics and special relativity, starting from the quantum mechanics of two coupled oscillators whose physics is thoroughly transparent to us.

Acknowledgments

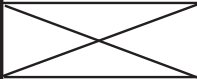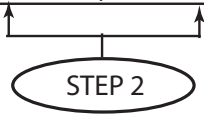| Scattering	Bound States	Space/Time
COMET	PLANET	
NEWTON		GALILEI
⊠	BOHR	
HEISENBERG, SCHRÖDINGER		
FEYNMAN	STEP 1	EINSTEIN
STEP 2		

Figure 6: History of dynamical and kinematical developments. It is important to note that mankind's unified understanding of scattering and bound states has been very brief. It is therefore not unusual to expect that separate theoretical models be developed for scattering and for bound states. The successes and limitations of the Feynman diagram are well known. If we cannot build a covariant quantum mechanics, it is worthwhile to see whether we can construct a relativistic theory of bound states to supplement quantum field theory, as Step 1 before attempting to construct a Lorentz-covariant theory applicable to both in Step 2.

thinking. I once asked him whether he thinks like Immanuel Kant. He said Yes. I then asked him whether Einstein was a Kantianist. He said firmly Yes, and told me he used to talk to Einstein while they were both at the Technical University of Berlin.

I asked him, in addition, whether he took a course in philosophy while he was a student. He said No. He became a Kantianist while doing physics. He then added that philosophers do not dictate people how to think, but they write down, sometimes systematically, the way in which people think. Kant was a very systematic person.

Professor Wigner then told me I was the first one to ask whether he was a Kantianist. He asked whether I took a course on Kant's philosophy. I said No. He then asked me how I came up with Kant's name. I told him that Kant is a very popular figure in Japan and Korea, presumably because Kantianism is very similar to the ancient Chinese philosophy known as Taoism [26, 27]. I told him also that Japan's Hideki Yukawa's thinking was based on Taoism [28].

After this conversation, I carried out my systematic study of Immanuel Kant and his philosophy, including my trip in 2005 to the Russian city of Kaliningrad, where Kant was born and spent eighty years of his entire life. I did my study in the way I do physics, without converting myself to a philosopher. Let us assume here that philosophy is a branch of physics.

If Kant was able to write down his philosophy based on observations he made in this world, Kaliningrad was his laboratory. How can one study Kant without visiting his laboratory?

With this background, I am very happy to say here that Richard Phillip Feynman was also a Kantianist. I would like to elaborate this point in this Appendix.

A   Feynman's Kantian Inclination

If Feynman felt that he was moving toward "One Physics" while writing papers on so many different subjects, it is because the same physics appears to him in different forms. Feynman had many different ways of looking at the same thing. We call this routinely Kantianism, or the philosophy formulated by Immanuel Kant (1724-1804).

Why Kant's philosophy so important in physics?. Is it right to talk about it in physics papers? First of all, Einstein's thinking was profoundly influ-

enced by Kant in his early years [8, 26]. Secondly, we can analyze Kant's thinking style with the methodology of physics, while avoiding philosophical jargons.

I came to the United States in 1954 right after my high-school graduation in Korea. I did my undergraduate study at the Carnegie Institute of Technology (now called Carnegie-Mellon University) and studied at Princeton University for my PhD degree in 1961. I have been on the physics faculty at the University of Maryland since 1962.

There is thus every reason to regard myself as an American physicist. Like all American physicists, I start writing papers if I do not have ideas. I gets publishable results while writing. On the other hand, I could not get rid of my philosophical background upon which my brain was configured during my childhood and my high-school years. Let us get back to this point in subsection. A.2.

Indeed, because of this background, I was able to raise the question of whether Feynman was a Kantianist. In this Appendix, I would like to explain what Kantianism is in the way physicists explain physics. I would then like to point out that Feynman was doing his physics in the way Kant was doing his philosophy.

A.1 Kantian Influence on Modern Physics

Unlike classical physics, modern physics depends heavily on observer's state of mind or environment. The importance of the observer's subjective viewpoint was emphasized by Immanuel Kant in his book entitled "Kritik der reinen Vernunft" whose first and second editions were published in 1781 and 1787 respectively.

The wave-particle duality in quantum mechanics is a product of Kantianism. If your detectors can measure only particle properties, particles behave like particles. On the other hand, if your detector can detect only wave properties, particles behave like waves. Heisenberg had come up with the uncertainty principle to reconcile these two different interpretations. This question is still being debated, and is a lively issue these days.

Furthermore, observers in different frames see the same physical system differently. Kant studied observations from moving frames extensively. However, using his own logic, he ended up with a conclusion that there must be the absolute inertial frame.

Einstein's special relativity was developed along Kant's line of thinking:

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things indeed depend on the frame from which observations are made. However, there is one big difference. Instead of the absolute frame, Einstein introduced an extra dimension, that is the Lorentzian world in which the time variable is integrated into the three spatial dimensions.

A.2 Kantianism and Taoism

I never had any formal education in oriental philosophy, but I know that my frame of thinking is affected by my Korean background. One important aspect is that Immanuel Kant's name is known to every high-school graduate in Korea, while he is unknown to Americans, particularly to American physicists. The question then is whether there is in Eastern culture a "natural frequency" which can resonate with one of the frequencies radiated from Kantianism developed in Europe.

I would like to answer this question in the following way. Koreans absorbed a bulk of Chinese culture during the period of the Tang dynasty (618-907 AD). At that time, China was the center of the world as the United States is today. This dynasty's intellectual life was based on Taoism which tells us, among others, that everything in this universe has to be balanced between its plus (or bright) side and its minus (or dark) side. This way of thinking forces us to look at things from two different or opposite directions. This aspect of Taoism could constitute a "natural frequency" which can be tuned to the Kantian view of the world where things depend how they are observed.

I would like to point out that Hideki Yukawa was quite fond of Taoism and studied systematically the books of Laotse and Chuangtse who were the founding fathers of Taoism [28]. Both Laotse and Chuangtse lived before the time of Confucius. It is interesting to note that Kantianism is also popular in Japan, and it is my assumption that Kant's books were translated into Japanese by Japanese philosophers first, and Koreans of my father's generation learned about Kant by reading the translated versions.

In 2005, when I went to Kaliningrad to study the origin of Kant's philosophy, I visited the Kant Museum twice. There was a room for important books written about Kant and his philosophy. There were many books written in Russian and in German. This is understandable because Kaliningrad is now a Russian city, and the Museum is under Russian management. Before 1945, Kaliningrad was a German city called Königsberg. Kant was born there and spent eighty years of his entire life there. He wrote all of his books

in German.

In addition, there are many books written in Japanese. Perhaps it could be a surprise to many people, but I was not. It only confirmed my Kantian background, as mentioned before. What is surprise even to me was that there are no books written in English in the Kant Museum. Again this did not surprise me. Americans are creative people in the tradition of Thomas Edison. However, Edison was not a Kantianist. I regard myself as an Edisonist and also as a Kantianist.

Let us see how Taoists can become Kantianists so easily. For Taoists, there are always two opposite faces of the same thing called "yang" (plus) and "ying" (minus). Finding the harmony between these two opposite points of view is the ideal way to live in this world. This is what Taoism is all about.

To Kantianists, however, it is quite natural for the same thing to appear differently in two different environments. The problem is to find the absolute value from these two different faces. Does this absolute value exist? According to Kant, it exists. To most of us, it is very difficult to find it if it exists.

Indeed, Kantianism is very similar to Taoism. It is very easy for a Taoist to become converted into a Kantianist. Let us see how Kant was influenced by the geography of the place where he spent his entire life. Let us see also how Taoism was developed in ancient China.

A.3 Geographical Origins of Kantianism and Taoism

Kant was born in the city of Königsberg. Since he spent eighty years of his entire life there, his mode of thinking was profoundly influenced by the lifestyle of Königsberg. Where is this city? Like the Mediterranean Ocean, the Baltic Sea was the basin of the civilization in its own area. About four hundred years ago, Lithuania and Poland were very strong countries. There was a Baltic costal area between these two countries, which became one of the commercial centers strong enough to assert independence from its two strong neighbors.

This place became a country called Prussia and became rich and strong enough to acquire a large area land west of Poland including Berlin. Then the center of gravity of Prussia moved to Germany, and the original Prussia became a province of Germany called "East Prussia." In Poland, this area is still called Prussia.

Königsberg was a coastal city in East Prussia, but after World War II,

East Prussia became divided into two parts and annexed to Poland and the Soviet Union. Könisberg is now a Russian city called Kaliningrad.

Let us look at the map containing both the Baltic and Black Seas. This area consists of Poland, Belarus, and Ukraine which are between Eastern and Western Europes. There are no natural barriers between this broad borderland, and anyone with a strong army could walk into or walk through this area. Accordingly the city of Königsberg had been under many different managements [29]. In addition, since the city was a commercial center like Venice, there were many visitors with financial power. The native citizens of Königsberg therefore had to entertain those powerful people with many different view points for the same thing. Kant wrote philosophy books based on the life style of his city.

Let us next how Taoism was developed in ancient China. After the last ice age, China consisted of many isolated pockets of population. Those people started moving toward the banks of China's two great rivers. This is how today's China started. When those Chinese came to the river banks with different languages with different customs, they were wise enough to realize that they had to live harmoniously with others. They started to communicate with others by drawing pictures, which later became the Chinese characters. Because they had different languages, they started singing to convey their feelings to others. This is the reason why spoken Chinese still has tones.

How about different thinkings? If one has an idea about something, there were others with different ideas. They simplified to two opposite ideas. If they were to live harmoniously, those two opposite views should exist together. They had to develop their philosophy based on "Plus and "Minus," and their balance. This is what the Taoism is all about.

What is striking is that both Kantianism and Taoism have their geographical origins. Both of them are based on the principle of accommodating different viewpoints. Modern physics takes into account the observer's environment and view points.

Modern physics was developed along the line of Kantian philosophy. If Feynman felt that he was looking for the same thing while writing papers on different subjects, he was a Kantianist.

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